6 Tools

6.1 Learning outcomes

After studying this text the learner should / should be able to:

1. Know, understand and be able to apply other yield measures.
2. Know, understand and be able to apply bond market tools like duration, Macauley duration and modified duration.
3. Appreciate the concept of convexity.
4. Appreciate the concept of LCC per basis point.
5. Understand the meaning of the yield curve and how it is derived.
6. Know and understand the various uses of the yield curve.
7. Understand the par yield curve and the zero-coupon yield curve and their uses.
8. Know the different shapes of the yield curve, what it represents in terms of short-term and long-term interest rates and expectations with regard to short-term interests rates (monetary policy).

6.2 Introduction

There are a number of tools that practitioners in the bond market employ daily to assist in operations. There are four tools / categories of such tools:

- Other yield measures.
- Duration.
- LCC per basis point
- The yield curve.

6.3 Other yield measures

6.3.1 Introduction

There are a number of other yield measures used in the bond market:

- Coupon.
- Yield to par.
- Simple yield to maturity.
- Current yield.
- Running yield.
- Holding period yield.
- Yield to call.

6.3.2 Coupon

Coupon is a yield measure that is applicable when a bond is issued at a rate (ytm) that matches the coupon. This means that the bond is issued at "par". Par means that the bond is issued at a price of 1.0 or 100%. We saw earlier that, generally, bonds are issued at a premium or a discount which means that the ytm is not equivalent to the coupon. In some countries bonds are issued at par.

6.3.3 Par yield

Par yield (or "yield to par") is an expression or yield measure that is applicable when a bond is trading at an ytm equal to the coupon rate.

6.3.4 Simple yield to maturity

The simple yield to maturity[38] is a measure used in some countries. It ignores reinvestment of the coupon and the capital loss or gain is amortised equally over the unexpired term to maturity. It is:

$$\text{Simple ytm} = \{cr + [(RP - CP) / t]\} / CP$$

where

cr = coupon rate pa
RP = redemption price at maturity (always = 1.0)
CP = clean price (reminder: all-in price – interest price)
t = time (unexpired years to maturity).

Example:

cr = 11.0% pa
CP = 0.97456
t = 8

$$\begin{aligned}\text{Simple ytm} &= \{0.11 + [(1.0 - 0.97456) / 8]\} / 0.97456 \\ &= [0.11 + (0.02544 / 8)] / 0.97456 \\ &= 0.11318 / 0.97456 \\ &= 11.613\% \text{ pa.}\end{aligned}$$

6.3.5 Current yield

The so-called *current yield* is a *simple measure* of return earned in a year. The current yield (cy) is the ratio of the coupon rate (cr) to the all-in market price (MP):

$$cy = cr / MP$$

Example:

cr = 12% pa
MP = 0.954554
cy = 0.12 / 0.954554
= 0.1257
= 12.57% pa.

It will be apparent that this measure ignores:

- reinvestment of coupon (compounding)
- any capital gain or loss, i.e. the difference between the all-in price paid and the redemption price at maturity (1.0); in the example it is a capital gain (1.0–0.054554).

6.3.6 Running yield

Running yield (ry) (also called *flat yield*) is the same as the current yield, except that the denominator is the clean price CP:

ry = cr / CP.

6.3.7 Holding period yield

Holding period yield (hpy) mathematics is similar to the ytm mathematics. While the ytm is the *average expected return* over the life of the bond, hpy is the yield for the period that a bond is held when sold before maturity. Formally it is the yield that equates the market price (MP) of the bond with the cash flows [coupon rate (cr) and the sale price (SP)] between the time of purchase and sale (n = holding period):

$$MP = [cr / (1 + hpy)^1] + [cr / (1 + hpy)^2] \dots [cr / (1 + hpy)^n] + [SP / (1 + hpy)^n].$$

All that remains is to solve for hpy. (Note that the assumptions here are that the bond is purchased on a coupon payment date and that the reinvestment rate of the coupons is equal to the coupon rate). In practice this is unlikely to be the case.

6.3.8 Yield to call[39]

Bonds with call options may never reach maturity; the issuer holds the option to retire them before maturity. The return measure *yield to call* (ytc) is regarded as a better measure of return than ytm in the case of call bonds. The calculation is the same as the ytm, but with two important differences:

- The anticipated call date is used as the maturity date (unlike non-call bonds, the call date can only be anticipated).
- The principal plus the call penalty (if applicable) is substituted for the principal.

An example is useful:

Face value (FV)	= LCC1 000 000
Term to maturity	= 10 years
Coupon	= 8.0% pa
Ytm now	= 9.0% pa
Price now	= LCC935 000
Anticipated call date	= 5 years
Anticipated penalty	= LCC50 000

$$ytc \quad = LCC935\ 000 \quad = [LCC80\ 000 / (1 + ytc)^1] \dots [LCC80\ 000 / (1 + ytc)^5] + [LCC1\ 050\ 000 / (1 + ytc)^5]$$

$$= 10.55\%.$$

6.4 Duration

6.4.1 Introduction

The concepts of duration and related measures are discussed under the following sections:

- Duration (price sensitivity).
- Modified duration.
- Convexity.
- LCC per basis point.

6.4.2 Duration

6.4.2.1 Introduction

Duration is an alternative approach to term to maturity. Because of the impact of the time to maturity and the coupon (etc.) on the price of bonds, an alternative index of maturity to calendar years was developed: *duration*. The basic idea was to create a *linear (= proportional) relationship between maturity and bond price volatility (or elasticity) irrespective of the level of the coupon rate*. In order to explain this we need again to go back to the basics.

The pricing of bonds was elucidated earlier with the assistance of an example. This is repeated here:

Settlement date:	30/9/2005
Maturity date:	30/9/2008
Coupon rate:	9% pa
Face value:	LCC1 000 000
Interest date:	30/9
ytm	8% pa.

In this example the cash flows occur as shown in Table 1.

Date	Coupon Payment	Face value	Compounding periods (cp)	Present value $C / (1 + ytm)^{cp}$
30/9/2006	LCC90 000	-	1	LCC83 333.33
30/9/2007	LCC90 000	-	2	LCC77 160.49
30/9/2008	LCC90 000	-	3	LCC71 444.90
30/9/2008	-	LCC1 000 000	3	LCC793 832.24
Total	**LCC270 000**	**LCC1 000 000**		**LCC1 025 770.96**
C = coupon. cp = compounding periods.				

Table 1: Cash flows and discounted values

The value of the bond is LCC1 025 770.96n and the price of the bond is 1.02577096 or 102.577096%. What was explicated in the table may be rewritten as (cr = coupon rate pa):

$$\text{Price} = [cr / (1 + ytm)^1] + [cr / (1 + ytm)^2] + [cr / (1 + ytm)^3] + [1 / (1 + ytm)^3].$$

This serves as an introduction to the characteristics of bonds which are explicated hereafter.

4.2.2 Impact of ytm change on price of bond

If the ytm on this bond (coupon = 9.0%) is increased to 11% pa (from 8.0%), the price is:

$$\begin{aligned}
\text{Price} &= [cr / (1 + ytm)^1] + [cr / (1 + ytm)^2] + [cr / (1 + ytm)^3] + [1 / (1 + ytm)^3] \\
&= (0.09 / 1.11^1) + (0.09 / 1.11^2) + (0.09 / 1.11^3) + (1 / 1.11^3) \\
&= (0.09 / 1.11) + (0.09 / 1.2321) + (0.09 / 1.367631) + (1 / 1.367631) \\
&= 0.08108108 + 0.07304602 + 0.06580722 + 0.73119138 \\
&= 0.95112570 \\
&= 95.112570\%.
\end{aligned}$$

Conclusion: if the rate rises, the price falls.

If the ytm on this bond (coupon = 9.0%) is decreased to 5% pa (from 8.0%), the price is:

$$\begin{aligned}
\text{Price} &= [cr / (1 + ytm)^1] + [cr / (1 + ytm)^2] + [cr / (1 + ytm)^3] + [1 / (1 + ytm)^3] \\
&= (0.09 / 1.05^1) + (0.09 / 1.05^2) + (0.09 / 1.05^3) + (1 / 1.05^3) \\
&= (0.09 / 1.05) + (0.09 / 1.1025) + (0.09 / 1.157625) + (1 / 1.157625) \\
&= 0.08571429 + 0.08163265 + 0.07774538 + 0.86383760 \\
&= 1.10892992. \\
&= 110.892992\%.
\end{aligned}$$

Conclusion: if the rate falls, the price rises.

This inverse relationship between ytm and price is portrayed in Figure 1.

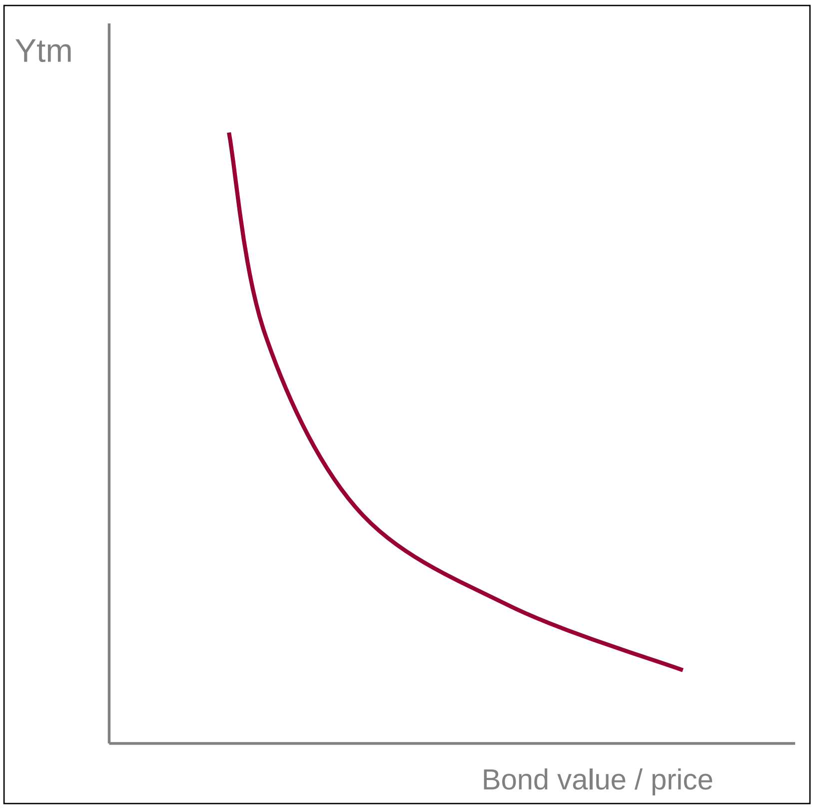


Figure 1: relationship between rate (ytm) and price (value)

In both cases the rate was changed by 300 bp. However, it will not be immediately apparent that the *price decline* in the case of the increased ytm was *smaller* than the *price increase* in the case of the lower ytm:

In the first case (ytm increase of 300bp): 1.02577096 – 0.95112570 = 0.07464526.

In the second case (ytm decrease of 300 bp): 1.10892992 – 1.02577096 = 0.08315896.

From these numbers one can also determine the *price elasticity* of the bond (i.e. the responsiveness of the bond's price to changes in ytm):

$$\text{Price elasticity} = (P_1 - P_0 / P_0) / (ytm_1 - ytm_0 / ytm_0)$$

where

P_1 = price at subsequent time
P_0 = price at initial time
ytm_1 = yield to maturity at subsequent time
ytm_0 = yield to maturity at initial time.

In the first case (ytm increase 8% to 11%):

$$\begin{aligned}\text{Price elasticity} &= (P_1 - P_0 / P_0) / (ytm_1 - ytm_0 / ytm_0)\\ &= (0.95112570 - 1.02577096 / 1.02577096) / (11\% - 8\% / 8\%)\\ &= (-0.07464526 / 1.02577096) / (3 / 8)\\ &= -0.07276991 / 0.375\\ &= -0.19405.\end{aligned}$$

In the second case (ytm decrease 8% to 5%):

$$\begin{aligned}\text{Price elasticity} &= (P_1 - P_0 / P_0) / (ytm_1 - ytm_0 / ytm_0)\\ &= (1.10892992 - 1.02577096 / 1.02577096) / (5\% - 8\% / 8\%)\\ &= (0.08315896 / 1.02577096) / (-3 / 8)\\ &= 0.08106972 / -0.375\\ &= -0.21619.\end{aligned}$$

For an upward movement in ytm from 8% to 11% the price elasticity for the bond is –0.19405, and a decline in ytm from 8% to 5% the price elasticity is –0.21619. Thus, the price elasticity is greater for the downward movement in interest rates than for an increase in interest rates.

Conclusion: ytm increases bring about proportionately smaller price changes than ytm decreases of the same magnitude.

6.4.2.3 Impact of term to maturity on bond price

If we use the same terms of the bond above (ytm = 8% pa; coupon = 9% pa), but increase the term from 3 to 4 years, the following is the price:

$$
\begin{aligned}
\text{Price} &= [cr / (1 + ytm)^1] + [cr / (1 + ytm)^2] + [cr / (1 + ytm)^3] + [cr / (1 + ytm)^4] + \\
&\quad [1 / (1 + ytm)^4] \\
&= (0.09 / 1.08^1) + (0.09 / 1.08^2) + (0.09 / 1.08^3) + (0.09 / 1.08^4) + \\
&\quad (1 / 1.08^4) \\
&= (0.09 / 1.08) + (0.09 / 1.166400) + (0.09 / 1.259712) + \\
&\quad (0.09 / 1.360489) + (1 / 1.360489) \\
&= 0.08333333 + 0.07716049 + 0.0714449 + 0.06615268 + 0.73502983 \\
&= 1.03312123 \\
&= 103.312123\%.
\end{aligned}
$$

If the ytm of this bond is increased to 11% pa (+300 bp – the same increase as in the case of an above example – for comparison purposes), its price will be:

$$\begin{aligned}
\text{Price} &= [cr / (1 + ytm)^1] + [cr / (1 + ytm)^2] + [cr / (1 + ytm)^3] + [cr / (1 + ytm)^4] + \\
&\quad [1 / (1 + ytm)^4] \\
&= (0.09 / 1.11^1) + (0.09 / 1.11^2) + (0.09 / 1.11^3) + (0.09 / 1.11^4) + \\
&\quad (1 / (1.11)^4) \\
&= (0.09 / 1.11) + (0.09 / 1.2321) + (0.09 / 1.367631) + \\
&\quad (0.09 / 1.518070) + (1 / 1.518070) \\
&= 0.08108108 + 0.07304602 + 0.06580722 + 0.0592858 + 0.65873115 \\
&= 0.93795127 \\
&= 93.795127\%.
\end{aligned}$$

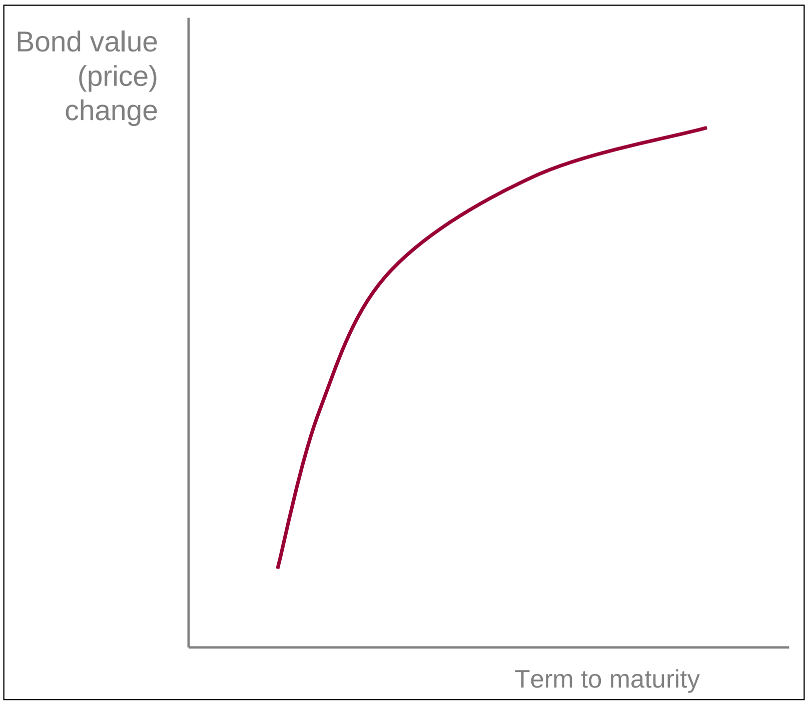


Figure 2: relationship between term to maturity and price change

The price change in the case of the 3-year bond with the coupon of 9%, but where the ytm increased by 300 bp (to 11%) was:

1.02577096 – 0.95112570 = 0.07464526 or 7.28% (or price elasticity of -0.1940).

In the case of the 4-year bond with the same characteristics, however, the price change was:

1.03312123 – 0.93795127 = 0.09516996 or 9.21% (or price elasticity of -0.2457).

Conclusion: the longer the bond the more price-sensitive it is to interest rate changes.

This relationship may be depicted as in Figure 2.

6.4.2.4 Impact of coupon rate on price of bond

Finally, we need to demonstrate that interest rate risk is inversely related to the coupon rate of the bond.

In the example of the bond above (i.e. 3-year, 9% coupon, and 8% ytm), the price was 1.02577096. When the ytm of the bond was increased to 11% pa, its price changed to 0.95112570. Thus, the change in the price was:

1.02577096 – 0.95112570 = 0.07464526 or 7.28% (or price elasticity of –0.199)

If we change to coupon to 20% and leave the other characteristics unchanged (i.e. term = 3 years and ytm = 8%), the price of this bond will be:

$$\begin{aligned}
\text{Price} &= [cr / (1 + ytm)^1] + [cr / (1 + ytm)^2] + [cr / (1 + ytm)^3] + [1 / (1 + ytm)^3] \\
&= (0.20 / 1.08^1) + (0.20 / 1.08^2) + (0.20 / 1.08^3) + (1 / 1.08^3) \\
&= (0.20 / 1.08) + (0.20 / 1.16640) + (0.20 / 1.259712) + (1 / 1.259712) \\
&= 0.18518519 + 0.17146776 + 0.15876645 + 0.79383224 \\
&= 1.30925164 \\
&= 130.9251643\%.
\end{aligned}$$

It the ytm on this bond (with a higher coupon) is lifted to 11% pa, the price changes to:

$$\begin{aligned}
\text{Price} &= [cr / (1 + ytm)^1] + [cr / (1 + ytm)^2] + [cr / (1 + ytm)^3] + [1 / (1 + ytm)^3] \\
&= (0.20 / 1.11^1) + (0.20 / 1.11^2) + (0.20 / 1.11^3) + (1 / 1.11)^3) \\
&= (0.20 / 1.11) + (0.20 / 1.2321) + (0.20 / 1.367631) + (1 / 1.367631) \\
&= 0.18018018 + 0.16232449 + 0.14623828 + 0.73119138 \\
&= 1.21993433 \\
&= 121.993433\%.
\end{aligned}$$

The price difference here is:

1.30925164 – 1.21993433 = 0.08931731 or 6.82% (or price elasticity of –0.1819).

Thus, compared with the 9% coupon bond, the 20% coupon bond exhibits a smaller price change when the ytm (rate) is increased to the same extent (i.e. 6.82% compared with 7.28%, or price elasticity of –0.1819 compared with –0.199).

Conclusion: interest rate risk increases inversely with coupon, ie the prices of higher coupon bonds are less sensitive to changes in interest rates than are the prices of lower coupon bonds. This may be depicted as in Figure 3.

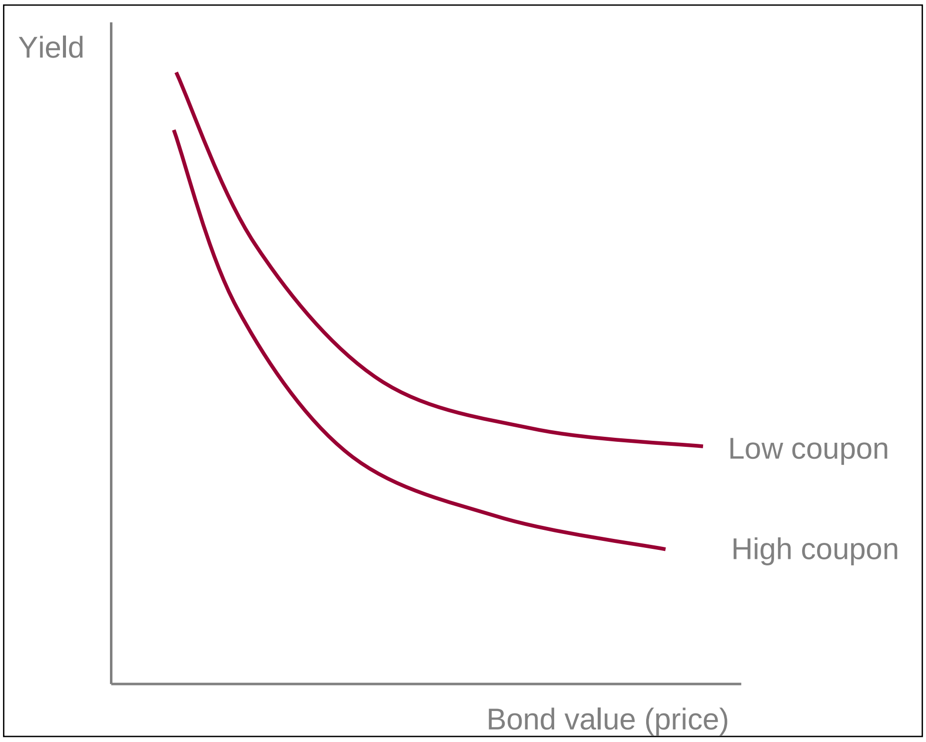


Figure 3: relationship between coupon and price

6.4.2.5 Macauley duration

We learned above that:

- Bond prices rise when rates (ytm) fall, and bond prices fall when rates increase.
- Ytm increases bring about proportionately smaller price changes than ytm decreases of the same magnitude.
- The maturity of a bond has an increasing effect on price sensitivity.
- Price risk increases inversely with coupon.

The above shows that, while maturity has a major impact on price (i.e. is a major determinant of interest rate risk), it is not the only determinant. Put another way, *maturity alone is not sufficient to measure interest rate risk*. Another maturity measure was required and the concept of *duration* was developed.

Duration was developed by Frederick Macaulay (see bibliography), and has been defined as the *effective maturity of a bond*, i.e. the *average maturity of a bond's promised cash flows* (i.e. coupons and principal). This maturity measure is a *gauge of interest rate sensitivity*.

Duration has also been described as a *method that converts a coupon bond (or any interest paying security) into its zero coupon equivalent*. By calculating the duration of all bonds in a portfolio, the bond portfolio manager is able to compare the interest rate sensitivity of all bonds held. The manager is essentially able to construct a *proportional (linear) relationship between maturity and price volatility*, irrespective of coupon rates. For example, if the portfolio manager doubles the duration of a bond portfolio s/he doubles the price elasticity of the portfolio.

Macaulay computed duration as the *weighted average of the times to each coupon or principal payment made by the bond*. Thus, duration is *the present value of interest and principal payments of a security weighted by the timing of those payments, divided by the present value of the security's stream of interest and principal payments*. The weight applied to each payment is the proportion of the total value of the bond accounted for by that payment, and the proportion is the present value of the payment divided by the bond price. An example is required (see Table 2).

Bond	Period to payment (years)	Payment amount (cash flow)	PV = payment discounted at 11% ytm	Weight* (PV as % of price)	Period to payment x weight = DURATION
LCC1 million 9% bond (annual interest)	1.0	90 000	81 081.08	0.086445	0.086445
	2.0	90 000	73 046.02	0.077878	0.155756
	3.0	90 000	65 807.22	0.070160	0.210480
	4.0	90 000	59 285.79	0.063208	0.252832
	4.0	1 000 000	658 730.97	0.701834	2.807336
			937 951.08	**1.000000**	**3.512849**
Zero coupon bond	1.0	-	-	-	-
	2.0	-	-	-	-
	3.0	-	-	-	-
	4.0	1 000 000	658 730.97	1.0	4.0
* weight = [CF / (1 + ytm)^{cp}] / bond price, where CF = cash flow; cp = compounding periods.					

Table 2: Duration example

As may be seen, the zero coupon bond has a duration equal to its term to maturity. This of course makes sense because a zero coupon bond has one payment – at the end of the life of the bond.

Thus *all zero coupon bonds are comparable in terms of interest rate sensitivity*. For example, the price of the 4-year zero coupon bond at an 11% ytm is 0.65873097. If the maturity date is doubled to 8 years, the price is 0.43392696, i.e. a price change of 34.13% [(0.65873097 – 0.43392696) / 0.65873097]. If the maturity of the bond is increased by another 4 years to 12 years, the price changes to 0.28584082, i.e. a change of 34.13% [(0.43392696 – 0.28584082) / 0.43392696]. Thus, the relationship between maturity and price is *linear* (i.e. proportional)

Similarly, all *coupon-paying bonds*, irrespective of coupon, that have a duration of 4 are comparable in terms of interest rate sensitivity, i.e. the extent of the price change, given the same extent of ytm change.

From the above it will be evident that the duration formula in the example used in the box above may be written as:

$$D = (\{[CF_1 / (1 + ytm)^1] / BP\} \times 1) + (\{[CF_2 / (1 + ytm)^2] / BP\} \times 2) + (\{[CF_3 / (1 + ytm)^3] / BP\} \times 3) + (\{[CF_4 / (1 + ytm)^4] / BP\} \times 4)$$

where

D = duration
CF = cash flow for periods 1 to 4
BP = bond price $[CF_1 / (1 + ytm)^1] + [CF_2 / (1 + ytm)^2] + [CF_3 / (1 + ytm)^3] + [CF_4 / (1 + ytm)^4]$.

An even easier way to calculate duration is shown in Table 3 (using the same numbers in the box above).

Period to payment (years)	Payment amount (cash flow)	PV of cash flows – at 11% ytm	time period	PV of cash flows x period
1.0	90 000	81 081.08	1.0	81 081.08
2.0	90 000	73 046.02	2.0	146 092.04
3.0	90 000	65 807.22	3.0	197 421.66
4.0	90 000	59 285.79	4.0	237 143.16
4.0	1 000 000	658 730.97	4.0	2 634 923.88
		937 951.08		**3 296 661.82**

Table 3: Duration example: LCC1 million, 9% coupon, 4-year bond

The duration of the bond is:

LCC3 296 661.82 / LCC937 951.08
= 3.51 (which ties with the above figure).

Thus, duration may also be written as:

$$D \quad = (\text{sum } PV_{cf} \times t) / \text{sum } PV_{cf}$$

where

D = duration
PV_{cf} = present value of each cash flow
t = time period (period to the cash flow).

In words, duration is calculated as: the sum of the present value of each cash flow times the applicable time period, divided by the sum of the present value of each cash flow (i.e. the PV of the bond).

6.4.2.6 The uses of duration

The uses of duration are:

- Useful statistic to calculate the *effective average maturity* of a portfolio.
- Tool for the *immunisation of a portfolio from market rate risk* (e.g. to protect the return from a portfolio against changes in rates; select securities with duration that match investment horizon).
- Measure of *interest rate sensitivity* of portfolios.

Because duration is related in a linear fashion to price volatility, there exists a useful relationship between *changes in interest rates (ytm) and percentage changes in prices*. This may be written as follows:

$$\Delta P / P = -D \times (\Delta ytm / 1 + ytm)$$

where

P = price
ΔP = change in price
D = duration
ytm = yield to maturity.

Example:

D = 4
Δytm = 100 bp
ytm = 9%

$$\begin{aligned}\Delta P / P &= -D \times (\Delta ytm / 1 + ytm)\\ &= -4 \times (0.01 / 1.09)\\ &= -4 \times 0.0091743\\ &= -0.036697\\ &= -3.67\%.\end{aligned}$$

This means that if a bond has a duration of 4, then for every 100 basis point change in the yield, the price will change by 3.67%. This would be the same for any other bond with a duration of 4, irrespective of coupon, term to maturity and rate level. Thus, bond prices change in an inversely proportional way according to duration.

6.4.3 Modified duration

In practice many portfolio managers make use of duration in a slightly different form, i.e. that of *modified duration* (i.e. modified from the Macaulay duration). This is as follows:

$$D_m = D / (1 + ytm)$$

where

D_m = modified duration
D = Macaulay duration
ytm = yield to maturity.

Using the above example:

$$\begin{aligned} D_m &= 4 / 1.09 \\ &= 3.66972. \end{aligned}$$

The formula used earlier [$\Delta P / P = -D \times (\Delta ytm / 1 + ytm)$] now changes to the following (i.e. in the case of a 100 basis point change in the yield):

$$\begin{aligned} \Delta P / P &= -D_m \times \Delta ytm \\ &= -3.66972 \times 1.0 \\ &= -3.67\%. \end{aligned}$$

In the case of a 1 basis point change in the yield:

$$\begin{aligned} \Delta P / P &= -D_m \times \Delta ytm \\ &= -3.66972 \times 0.01 \\ &= -0.0367\%. \end{aligned}$$

This means that the percentage change in the price of a bond is *approximately* equal to the product of modified duration and the change in the yield of the bond. Thus, for each 100 basis point change in the yield of a bond with a modified duration of 3.67, the price will change by 3.67%. It follows that for each basis point change in the ytm of bonds with the same duration, the price will change by 0.0367%.

6.4.4 Convexity

The word "approximately" was used in italics above because duration correctly measures price sensitivity of bonds only for *small changes in interest rates*. With large changes in rates, duration becomes a less accurate measure of price changes. The reason for this is illustrated in Figure 4. *The true relationship between prices and yields is convex and not linear* (this can be proven empirically). Thus, with each large rate *increase* from the rate prevailing now (the intersection of the linear line and the curve) the duration model overestimates the *fall* in the price of the bond. Conversely, for large rate decreases, the duration model underestimates the increase in the price.

It will be clear that the duration model always underestimates the value (price) of the bond after large changes in interest rates (either positive or negative).

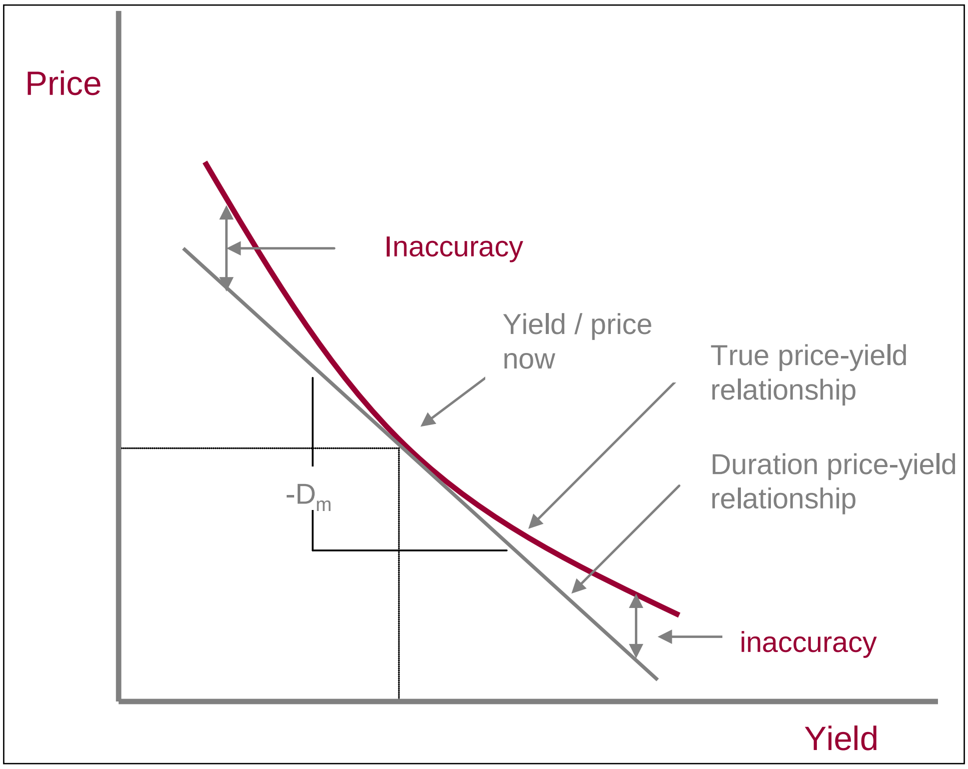


Figure 4: convexity

6.5 LCC per basis point

LCC per basis point (LCCbp) is simply the amount of LCC per basis point change in the rate on LCC1 million nominal value of the relevant bond, for example from 10.03% pa (ytm) to 10.02% pa (ytm).

This number is an important gauge for traders in the bond market in terms of assessing potential profits or losses. Thus, if the rate (ytm) on a R186 bond changes from 7.89% to 7.88% the LCC amount per LCC1 million nominal value of the bond (in this case a profit) may be LCC500. Clearly if the bond holding is LCC10 million, then the LCCbp = LCC5 000.

The LCCbp differs from bond to bond, and depends on coupon and term to maturity.

6.6 The yield curve (term structure of interest rates)

6.6.1 Introduction

This section has to do with the rates on bonds of various remaining terms to maturity at a point in time, i.e. the relationship between bond rates and terms to maturity, called the *term structure of interest rates* and the *yield curve*. We present a *positively-sloped* (or *normal*) yield curve in Figure 5.

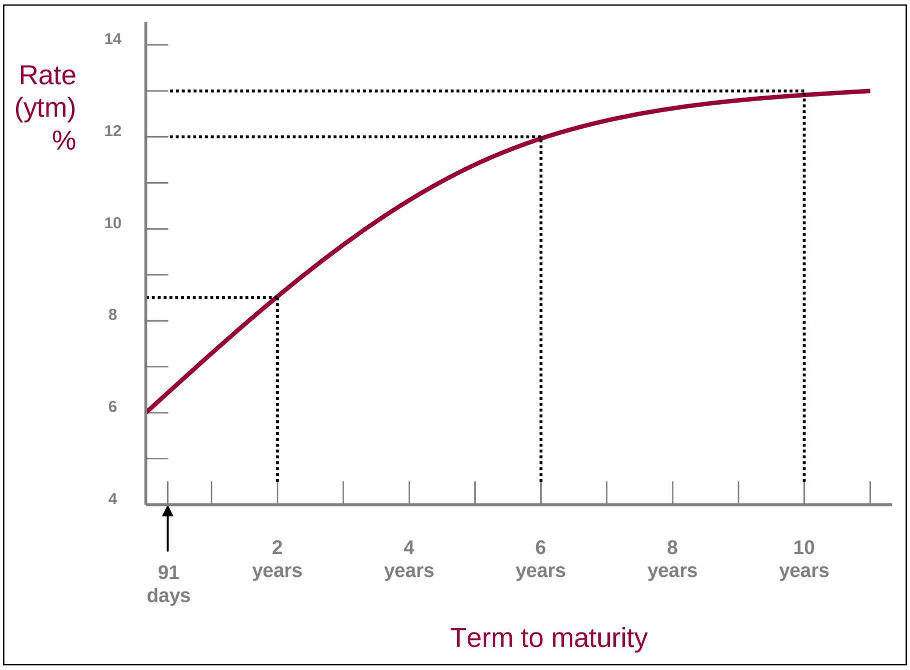


Figure 5: normal yield curve

Let us assume that this is a yield curve for government securities (treasury bill rate and bond rates[40]) at 4pm on 20 June 2009. The yield curve is telling us that the rates shown in Table 7.1 were recorded on that day (Note: they are read from the curve).

MATURITY OF SECURITY	RATE
91-days (treasury bill)	6.5%
1 year (government bond)	7.5%
2 years (government bond)	8.5%
3 years (government bond)	9.65%
4 years (government bond)	10.60%
5 years (government bond)	11.42%
6 years (government bond)	12.00%
7 years (government bond)	12.32%
8 years (government bond)	12.50%
9 years (government bond)	12.81%
10 years (government bond)	13.00%
11 years (government bond)	13.11%

Table 4: government security rates recorded on 20 June 2009

Where did this yield curve come from? It was constructed from the rates that prevailed on government securities of various maturities at 4pm on 20 June 2009. Figure 6 depicts this.

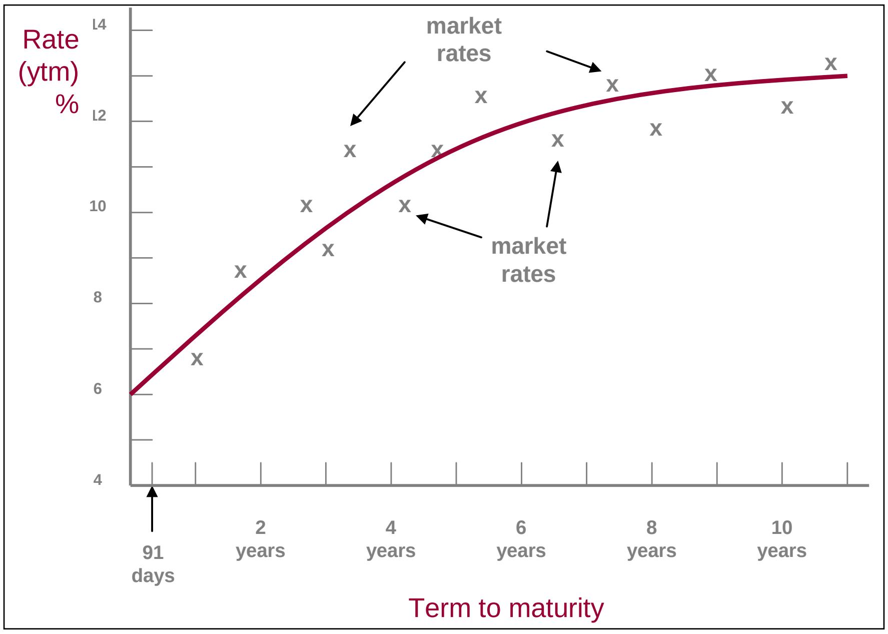


Figure 6: market rates and constructed yield curve

The market rates on government securities of different maturities are represented by the x's and the yield curve constructed and drawn with the use of sophisticated statistical techniques. Thus, it will be apparent that the yield curve is a *graphical representation of the relationship between rate and term to maturity of bonds.*

A yield curve is a useful tool:

- Rates for year intervals can be derived for analysis purposes. For example, rates can be derived from the curve for 1 year, 2 years, 3 years, etc. Thus, over a period of time a series of rates for various terms is available. Recording the rate on a *specific* 10-year bond is of no use because each month the bond has one month less to maturity (i.e. it is no longer a 10-year bond).
- Securities can be valued using the curve. The holder of a poorly traded bond is able to value the bond because the curve gives the "average" rate for all terms.
- The curve serves as a benchmark for both buyers of bonds and new issues of bonds.

It will be evident that in a sophisticated market the points (the x's) will not be as scattered as in the above example; they will be closer to the curve that is constructed from them.

It is to be noted that the above discussion was concerned with the *yield to maturity (ytm) yield curve*. It is the most familiar yield curve and is a representation of the relationship between yield to maturity and term to maturity of a group of homogenous securities (usually government).

6.6.2 Disadvantage of the ytm yield curve

There is a "problem" with the ytm yield curve. In the definition of ytm is the implicit assumption that coupon payments are reinvested at the ytm; this is rarely achieved (which can be called reinvestment risk). The only bond devoid of reinvestment risk is the zero coupon bond that has one payment at the end of its life.[41] For these reasons other yield curve types have been devised.

6.6.3 Par yield curve

As noted, the coupon rate has an effect on the price sensitivity of bonds. For this reason, various markets make use of a *par yield curve*. This is a yield curve of rates on bonds the prices of which are close to or at par (100% – at par ytm = coupon), which means that the effect of coupon on price is eliminated. This makes the various points on the yield curve comparable.

A caveat is required here. The par yield curve is more relevant in countries where bonds are traded on a clean price basis (accrued interest is taken into account after the deal is done). In South Africa dealing takes place on a yield basis; consequently, when ytm = coupon, the price is not necessarily 100%. The price equals 100% only on coupon dates.

6.6.4 Coupon yield curve

The coupon yield curve is a representation of ytm and term to maturity of a group of homogenous bonds that have the same coupon. Generally the high coupon bonds trade at higher rates than low coupon bonds (i.e. are valued lower), and this is so for two main reasons[42]:

- Reinvestment risk. It is likely that rates will fall during the life of all bonds; bonds with high coupons are prejudiced in relation to low coupon bonds because the coupons are invested at lower rates.
- Tax. High-rate taxpayers prefer low coupon bonds because capital gains are higher than in the case of high coupon bonds, capital gains tax is usually lower, and the tax on capital gains is deferred.

Yield curves of same coupon, homogenous (i.e. same credit quality) bonds are not constructed and compared by investors in order to gain from yield anomalies that may exist (this applies to zero / spot versus ytm curves). They are merely of academic interest: they (the differential) signify one of the major disadvantages of the ytm curve, and generally arise as a result of the mismatch in the demand and supply of bonds in the longer term maturity sector. Insurers, in order to match longer term liabilities, demand longer dated bonds and favour low coupon bonds for the abovementioned reasons. A consequence of this is (in many countries) the widening of the differential between low and high coupon bonds in the long end of the maturity scale.[43]

6.6.5 Yield curve of "on-the-run treasury issues"

In the US, financial market participants in this regard talk of the yield curve of "*on-the-run treasury issues*". This yield curve is constructed from the most recently issued treasury bonds, notes and bills (the notes and bonds are issued at a price of 100%). This yield curve is a proxy for a par yield curve.

6.6.6 Spot (zero-coupon) yield curve[44]

The ideal or "pure" yield curve is a *zero-coupon yield curve* (also called *spot yield curve*), i.e. a curve constructed from the rates on a series of central government zero coupon bonds and treasury bills. This means that the term of each maturity matches its duration and the rates are comparable. The problem is that in most markets zero coupon bonds are rare or non-existent; consequently, the curve has to be derived from coupon bond yields.

Spot yields satisfy the equation [assumptions: annual coupons; calculation takes place on a coupon payment date (therefore no accrued interest)]:

$$\begin{aligned} \text{AIP} &= \sum_{t=1}^{T} \left[c / (1 + sy_t)^t\right] + [\text{PVB} / (1 + sy_T)^T \\ &= \sum_{t=1}^{T} c \cdot DF_t + \text{PVB} \cdot DF_T \end{aligned}$$

where

AIP = all-in price of bond (dirty price)
c = coupon (annual and fixed)
sy_t = spot (zero coupon) yield with t years to maturity
PVB = par value of bond
DF_t = $1 / (1 + si_t)^t$ = corresponding discount factor.
Clearly, sy_1 = current 1-year spot yield; sy_2 = current 2-year spot yield; sy_3 …..

6.6.7 Shape of yield curve

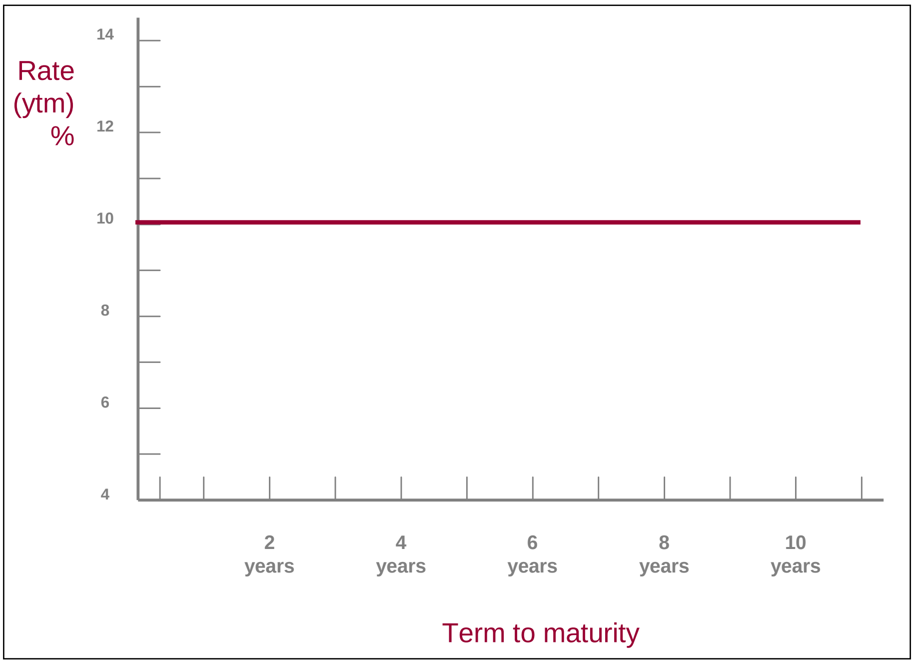


Figure 7: flat yield curve

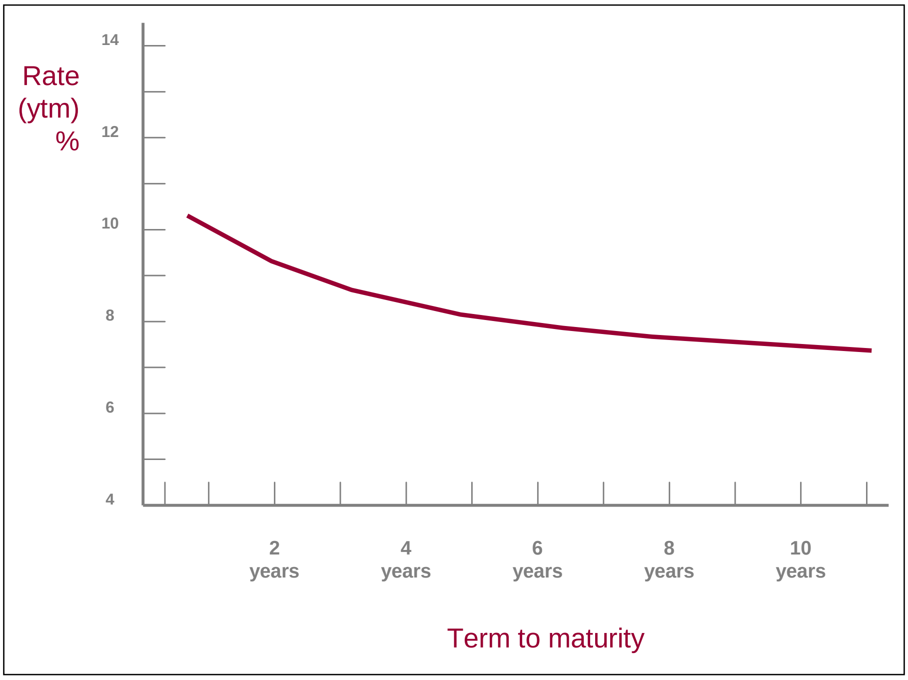


Figure 8: flat yield curve

Yield curves take on different shapes at different times. The normal curve is the one presented in the examples above, i.e. it is *positively sloped*, and it implies that the longer the bond the higher the return. Investors are rewarded for holding bonds of longer maturity. The other two basic shapes are the *flat yield curve* and the *inverted or negatively sloped yield curve*. The flat curve is portrayed in Figure 7.

The flat yield curve implies that there is no reward for the risk of a longer-term investment. Irrespective of term to maturity, all investors in government bonds earn a rate (ytm) of 9.4% pa in this example. This curve usually represents the stage between normal and inverse and vice versa.

The inverted or negatively sloped yield curve is illustrated in Figure 8. This curve tells us that investors are negatively compensated for holding long-term securities; they are "prejudiced" in relation to the holders of short-term securities – or so it appears. In reality, this yield curve normally comes about in periods of high rates when the monetary authorities are conducting a severe and tight monetary policy, driving up short-term rates. The long-term investors are content to accept short rates being higher than long rates because they *harbour strong expectations* that the shape of the yield curve is about to change to a normal shape and that the entire curve will shift downwards.

This means that the inverse yield curve is indicating that longer term investors are willing to accept lower rates now in exchange for large expected capital gains in the near future, i.e. the *income given up will be more than compensated for by the capital gain*.

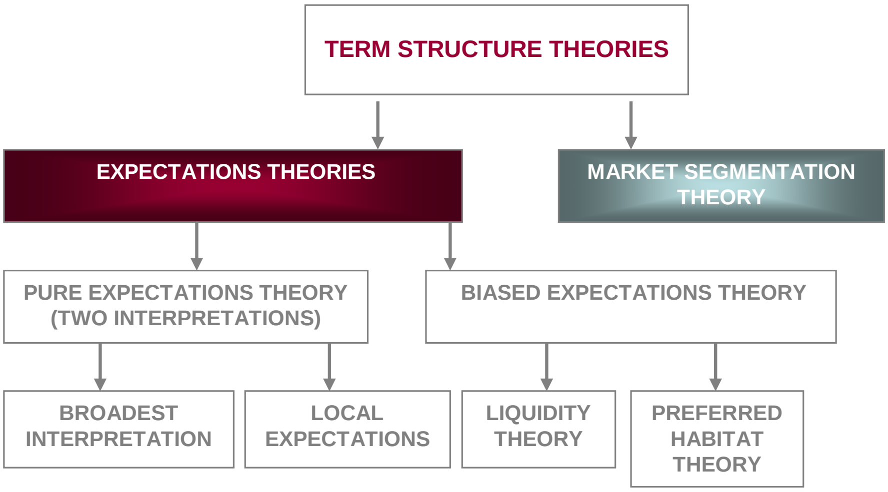
6.6.8 Theories of the term structure of interest rates

Two main theories have evolved to explain the yield curve, i.e. the expectations theory and the market segmentation theory. The former is categorised[45] into the pure expectations theory (of which there are two interpretations) and the biased expectations theory. There are two interpretations of the latter: the liquidity theory and the preferred habitat theory. Box 1 presents the term structure theories.

All these theories share a hypothesis about the behaviour of short-term forward rates and assume that the forward rates in current long-term bond rates are closely related to market participants' *expectations* about the future short-term rates.

The *pure expectations theory* postulates that the yield curve at any point in time (i.e. forward rates) reflects the market's expectations of future short-term rates. Thus, an investor with a 10-year investment horizon has a choice of buying a 10-year bond (and earn the current yield on his bond) or buying 10 successive 1-year bonds. The return on the two investments will be the same, i.e. long-term rates are geometric averages of current and expected future short-term rates.

In terms of this theory, a positively shaped yield curve indicates that short-term rates will rise over the investment term, and a flat curve indicates that short rates are to be stable over the investment horizon.



Box 1: term structure theories

As noted, there are basically *two broad interpretations* of this theory. The main criticism of this theory is that it does *not consider the risks* associated with investing in bonds.

The *liquidity theory* suggests that investors will hold longer term securities only if they are offered a long-term rate that is higher than the average of expected future rates by a risk premium that is *positively related* to the term to maturity (i.e. rises uniformly with maturity). Put another way: *the expected return from holding a series of short bonds is lower than the expected return from holding a long-bond over the same time period*. Thus, forward rates are not an unbiased estimate of the market's expectations of future rates because they embody a liquidity premium.

The *preferred habitat theory* buys the theory that the term structure of interest rates reflects the expectation of the future path of interest rates and the risk premium. However, it rejects the notion that the risk premium must rise uniformly with maturity. Thus, the risk premium can be positive or negative and can induce investors to move out of their preferred habitat, i.e. their preferred part of the curve. It will be evident that in terms of this theory the yield curve can be positively sloping, inverse or flat.

The *market segmentation theory* holds that investors have preferred maturities of bonds dictated by their liabilities. Thus, banks will hold short-term securities and pension funds / insurers long-term securities. They will not shift from one sector to another to take advantage of opportunities. The yield curve reflects supply and demand conditions in the various maturity sectors of the yield curve.

6.7 Summary

There are number of tools that have been developed by bond market participants over the past few decades: alternative yield measures, duration, LCC per basis point and the yield curve. The alternative yield measures offer quick guides to returns. Duration is an alternative measure to term to maturity and is useful in terms of price-sensitivity comparisons.

LCC per basis point enables dealers / speculators to gauge the risk and return parameters of positions. The yield curve is a representation of the relationship between rate and term to maturity at a point in time; it is an extremely useful tool in bond market analysis.

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